A-Level Mathematics Transition Booklet

The Abbey School
Maths Department

What is this booklet for?

This Transition work is designed to help you to bridge the gap between your GCSE studies and AS/A Level.

Why do Transition work?

Preparation is crucial for studying A levels. A levels require you to be an independent learner. Although you have fewer subjects, A levels require different study skills and the volume of work is greater due to the increased demand of depth and detail. The exercises in this booklet will ensure that you are ready for the exciting challenges of becoming an A level student in September.

Information about the course and exams

Exam Board	Edexcel
Examinations:	For A Level, you sit all three papers at the end of year 2 (Year 13): 2 x Pure (2 hours), 1 x Statistics/Mechanics (2 hours)
Lessons:	9 lessons per fortnight ; 6 lessons Pure and 3 Applied lessons (Statistics/Mechanics)

Preparation ahead of September

The amount of time required for the Transition work included in this booklet is not long. **We recommend significantly more time to be spent during the summer to ensure you come properly prepared.** Naturally, the harder you found the GCSE syllabus, the more time you are likely to need to put in. It is crucial you have a sound understanding and fluency on the following as you will regularly be using these skills:

Pure	Statistics/Mechanics
Factorising/solving quadratics	Venn diagrams/set notation
Difference of two squares	Cumulative Frequency/Box plots
Laws of indices	Interquartile range vs range
Fractional and negative indices	Averages from frequency tables

Surds	Probability
Inequalities	Sample space and tree diagrams
Graphs and transformation of graphs	Histograms
Trigonometry	Interpreting data
Simultaneous equations (elimination/substitution)	Commenting on results and trends
Changing subject of formula	
Algebraic fractions	
Calculations with decimals/negatives/fractions	
Gradients and equations of straight lines	
Circle theorems	

You might find your GCSE textbook to be sufficient to go over above, alternatively look ahead using the A level textbooks (see next page). You may also find the following resource helpful:

Useful websites
Throughout the course, the following websites are regularly used. You may find them useful as you prepare over the summer.

www.examsolutions.net
www.physicsandmathstutor.com
www.mathsgenie.co.uk
www.mymaths.co.uk
www.mathspapers.com
https://alevelmathsrevision.com/bridging-the-gap/
https://www.pearson.com/en-gb/schools/subject-resources/mathematics/unrivalled-support/support-from-pearson/gcse-maths-transition-to-a-level.html

Completing the square*

Key points

- Completing the square for a quadratic rearranges $ax^2 + bx + c$ into the form $p(x + q)^2 + r$
- If $a \neq 1$, then factorise using a as a common factor.

Examples

Example 1   Complete the square for the quadratic expression $x^2 + 6x - 2$

$x^2 + 6x - 2$ $= (x + 3)^2 - 9 - 2$ $= (x + 3)^2 - 11$	**1** Write $x^2 + bx + c$ in the form $$\left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$$ **2** Simplify

Example 2   Write $2x^2 - 5x + 1$ in the form $p(x + q)^2 + r$

$2x^2 - 5x + 1$ $= 2\left(x^2 - \frac{5}{2}x\right) + 1$ $= 2\left[\left(x - \frac{5}{4}\right)^2 - \left(\frac{5}{4}\right)^2\right] + 1$ $= 2\left(x - \frac{5}{4}\right)^2 - \frac{25}{8} + 1$ $= 2\left(x - \frac{5}{4}\right)^2 - \frac{17}{8}$	**1** Before completing the square write $ax^2 + bx + c$ in the form $$a\left(x^2 + \frac{b}{a}x\right) + c$$ **2** Now complete the square by writing $x^2 - \frac{5}{2}x$ in the form $$\left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$$ **3** Expand the square brackets – don't forget to multiply $\left(\frac{5}{4}\right)^2$ by the factor of 2 **4** Simplify

Practice

1 Write the following quadratic expressions in the form $(x + p)^2 + q$

    a $x^2 + 4x + 3$ **b** $x^2 - 10x - 3$

    c $x^2 - 8x$ **d** $x^2 + 6x$

    e $x^2 - 2x + 7$ **f** $x^2 + 3x - 2$

2 Write the following quadratic expressions in the form $p(x + q)^2 + r$

    a $2x^2 - 8x - 16$ **b** $4x^2 - 8x - 16$

    c $3x^2 + 12x - 9$ **d** $2x^2 + 6x - 8$

3 Complete the square.

    a $2x^2 + 3x + 6$ **b** $3x^2 - 2x$

    c $5x^2 + 3x$ **d** $3x^2 + 5x + 3$

Extend

4 Write $(25x^2 + 30x + 12)$ in the form $(ax + b)^2 + c$.

Circle theorems

Key points

- A chord is a straight line joining two points on the circumference of a circle.
  So AB is a chord.

- A tangent is a straight line that touches the circumference of a circle at only one point.
  The angle between a tangent and the radius is 90°.

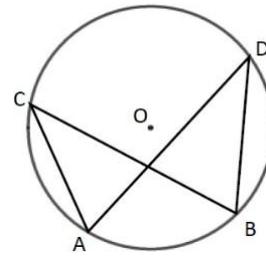- Two tangents on a circle that meet at a point outside the circle are equal in length.
  So AC = BC.

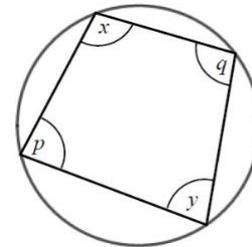- The angle in a semicircle is a right angle.
  So angle ABC = 90°.

- When two angles are subtended by the same arc, the angle at the centre of a circle is twice the angle at the circumference.
  So angle AOB = 2 × angle ACB.

- Angles subtended by the same arc at the circumference are equal. This means that angles in the same segment are equal.
  So angle ACB = angle ADB and angle CAD = angle CBD.

- A cyclic quadrilateral is a quadrilateral with all four vertices on the circumference of a circle. Opposite angles in a cyclic quadrilateral total 180°.
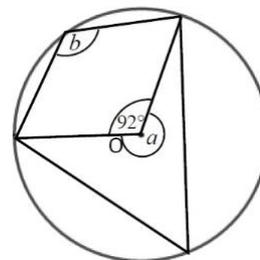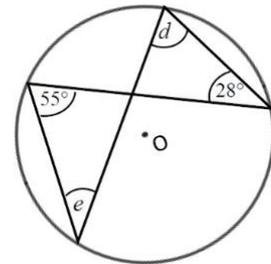 So $x + y = 180°$ and $p + q = 180°$.

- The angle between a tangent and chord is equal to the angle in the alternate segment, this is known as the alternate segment theorem.
  So angle BAT = angle ACB.

Examples

Example 1 Work out the size of each angle marked with a letter.
Give reasons for your answers.

Angle $a = 360° − 92°$ $\quad = 268°$ as the angles in a full turn total 360°.	**1** The angles in a full turn total 360°.
Angle $b = 268° ÷ 2$ $\quad = 134°$ as when two angles are subtended by the same arc, the angle at the centre of a circle is twice the angle at the circumference.	**2** Angles a and b are subtended by the same arc, so angle b is half of angle a.

Example 2 Work out the size of the angles in the triangle.
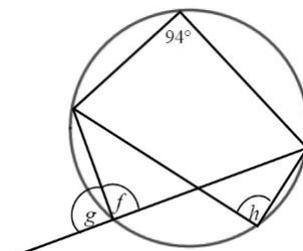Give reasons for your answers.

Angles are 90°, 2c and c.	**1** The angle in a semicircle is a right angle.
$90° + 2c + c = 180°$	**2** Angles in a triangle total 180°.
$90° + 3c = 180°$	**3** Simplify and solve the equation.
$3c = 90°$	
$c = 30°$	
$2c = 60°$	
The angles are 30°, 60° and 90° as the angle in a semi-circle is a right angle and the angles in a triangle total 180°.	

Example 3 Work out the size of each angle marked with a letter.
Give reasons for your answers.

Angle d = 55° as angles subtended by the same arc are equal.	**1** Angles subtended by the same arc are equal so angle 55° and angle d are equal.
Angle e = 28° as angles subtended by the same arc are equal.	**2** Angles subtended by the same arc are equal so angle 28° and angle e are equal.

Example 4 Work out the size of each angle marked with a letter.
Give reasons for your answers.

Angle f = 180° − 94°	**1** Opposite angles in a cyclic quadrilateral total 180° so angle 94° and angle f total 180°.
= 86°	
as opposite angles in a cyclic quadrilateral total 180°.	
	(continued on next page)

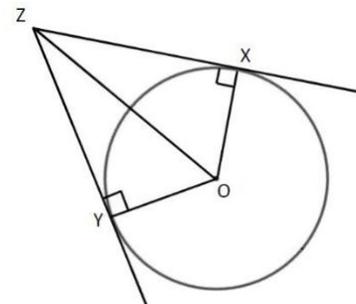| Angle $g = 180° - 86°$ $\qquad = 84°$ as angles on a straight line total 180°. Angle $h =$ angle $f = 86°$ as angles subtended by the same arc are equal.	**2** Angles on a straight line total 180° so angle f and angle g total 180°. **3** Angles subtended by the same arc are equal so angle f and angle h are equal.

Example 5   Work out the size of each angle marked with a letter.
 Give reasons for your answers.

Angle $i = 53°$ because of the alternate segment theorem. Angle $j = 53°$ because it is the alternate angle to 53°. Angle $k = 180° - 53° - 53°$ $\qquad = 74°$ as angles in a triangle total 180°.	**1** The angle between a tangent and chord is equal to the angle in the alternate segment. **2** As there are two parallel lines, angle 53° is equal to angle j because they are alternate angles. **3** The angles in a triangle total 180°, so $i + j + k = 180°$.

Example 6   XZ and YZ are two tangents to a circle with centre O.
 Prove that triangles XZO and YZO are congruent.

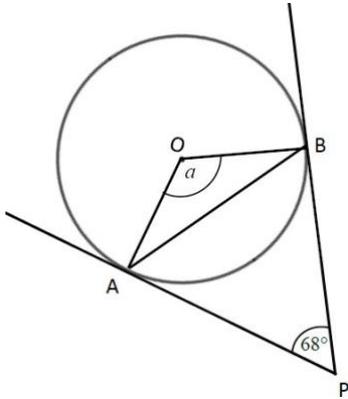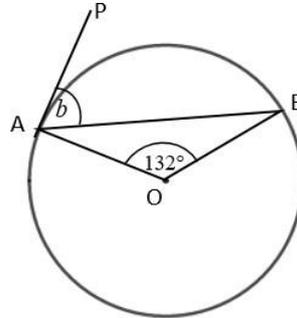| Angle OXZ = 90° and angle OYZ = 90° as the angles in a semicircle are right angles. OZ is a common line and is the hypotenuse in both triangles. OX = OY as they are radii of the same circle. So triangles XZO and YZO are congruent, RHS.	For two triangles to be congruent you need to show one of the following. • All three corresponding sides are equal (SSS). • Two corresponding sides and the included angle are equal (SAS). • One side and two corresponding angles are equal (ASA). • A right angle, hypotenuse and a shorter side are equal (RHS).

Practice

1   Work out the size of each angle marked with a letter.
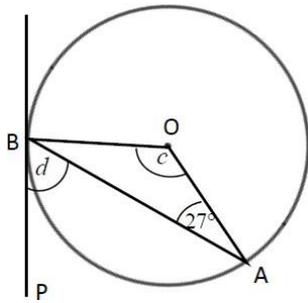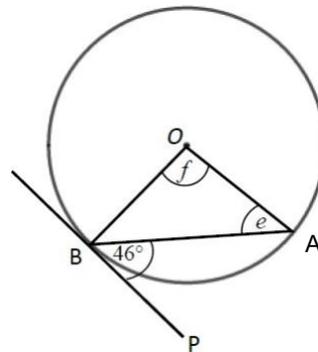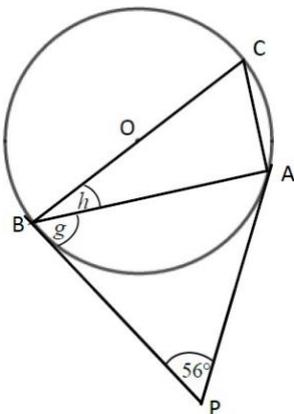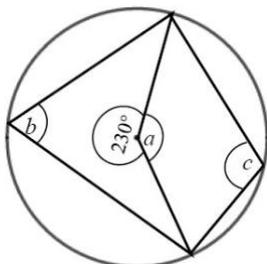   Give reasons for your answers.

a

b

c

d

e

2   Work out the size of each angle marked with a letter.
    Give reasons for your answers.

a

b

c

d

3   Work out the size of each angle marked with a letter.
    Give reasons for your answers.

a

b

c

d

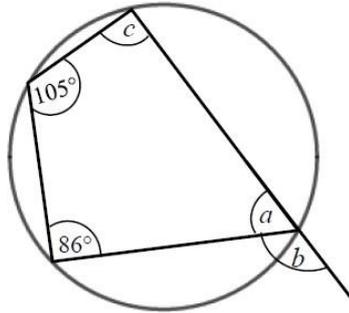**4**   Work out the size of each angle marked with a letter.
          Give reasons for your answers.

a

> **Hint**
>
> An exterior angle of a
> cyclic quadrilateral is
> equal to the opposite
> interior angle.

b   **c**
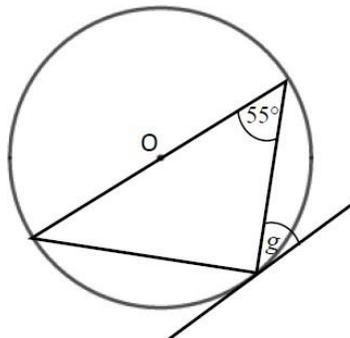

d

> **Hint**
>
> One of the angles
> is in a semicircle.

Extend

5   Prove the alternate segment theorem.

Solving quadratic equations by factorisation*

Key points

- A quadratic equation is an equation in the form $ax^2 + bx + c = 0$ where $a \neq 0$.
- To factorise a quadratic equation find two numbers whose sum is b and whose products is ac.
- When the product of two numbers is 0, then at least one of the numbers must be 0.
- If a quadratic can be solved it will have two solutions (these may be equal).

Examples

Example 1   Solve $5x^2 = 15x$

$5x^2 = 15x$	**1** Rearrange the equation so that all of the terms are on one side of the equation and it is equal to zero.
$5x^2 - 15x = 0$	Do not divide both sides by x as this would lose the solution $x = 0$.
$5x(x - 3) = 0$	**2** Factorise the quadratic equation. $5x$ is a common factor.
So $5x = 0$ or $(x - 3) = 0$	**3** When two values multiply to make zero, at least one of the values must be zero.
Therefore $x = 0$ or $x = 3$	**4** Solve these two equations.

Example 2   Solve $x^2 + 7x + 12 = 0$

$x^2 + 7x + 12 = 0$	**1** Factorise the quadratic equation. Work out the two factors of $ac = 12$ which add to give you $b = 7$.
$b = 7$, $ac = 12$	(4 and 3)
$x^2 + 4x + 3x + 12 = 0$	**2** Rewrite the b term ($7x$) using these two factors.
$x(x + 4) + 3(x + 4) = 0$	**3** Factorise the first two terms and the last two terms.
$(x + 4)(x + 3) = 0$	**4** $(x + 4)$ is a factor of both terms.
So $(x + 4) = 0$ or $(x + 3) = 0$	**5** When two values multiply to make zero, at least one of the values must be zero.
Therefore $x = -4$ or $x = -3$	**6** Solve these two equations.

Example 3   Solve $9x^2 - 16 = 0$

$9x^2 - 16 = 0$ $(3x + 4)(3x - 4) = 0$ So $(3x + 4) = 0$ or $(3x - 4) = 0$ $x = -\dfrac{4}{3}$ or $x = \dfrac{4}{3}$	**1** Factorise the quadratic equation. This is the difference of two squares as the two terms are $(3x)^2$ and $(4)^2$. **2** When two values multiply to make zero, at least one of the values must be zero. **3** Solve these two equations.

Example 4   Solve $2x^2 - 5x - 12 = 0$

$b = -5,\ ac = -24$ So $2x^2 - 8x + 3x - 12 = 0$ $2x(x - 4) + 3(x - 4) = 0$ $(x - 4)(2x + 3) = 0$ So $(x - 4) = 0$ or $(2x + 3) = 0$ $x = 4$ or $x = -\dfrac{3}{2}$	**1** Factorise the quadratic equation. Work out the two factors of $ac = -24$ which add to give you $b = -5$. $(-8$ and $3)$ **2** Rewrite the b term $(-5x)$ using these two factors. **3** Factorise the first two terms and the last two terms. **4** $(x - 4)$ is a factor of both terms. **5** When two values multiply to make zero, at least one of the values must be zero. **6** Solve these two equations.

Practice

1   Solve

    a $6x^2 + 4x = 0$ **b** $28x^2 - 21x = 0$

    c $x^2 + 7x + 10 = 0$ **d** $x^2 - 5x + 6 = 0$

    e $x^2 - 3x - 4 = 0$ **f** $x^2 + 3x - 10 = 0$

    g $x^2 - 10x + 24 = 0$ **h** $x^2 - 36 = 0$

    i $x^2 + 3x - 28 = 0$ **j** $x^2 - 6x + 9 = 0$

    k $2x^2 - 7x - 4 = 0$ **l** $3x^2 - 13x - 10 = 0$

2   Solve

    a $x^2 - 3x = 10$ **b** $x^2 - 3 = 2x$

    c $x^2 + 5x = 24$ **d** $x^2 - 42 = x$

    e $x(x + 2) = 2x + 25$ **f** $x^2 - 30 = 3x - 2$

    g $x(3x + 1) = x^2 + 15$ **h** $3x(x - 1) = 2(x + 1)$

> **Hint**
>
> Get all terms onto one side of the

Solving quadratic equations by completing the square*

A LEVEL LINKS
Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- Completing the square lets you write a quadratic equation in the form $p(x + q)^2 + r = 0$.

Examples

Example 5   Solve $x^2 + 6x + 4 = 0$. Give your solutions in surd form.

$x^2 + 6x + 4 = 0$	1 Write $x^2 + bx + c = 0$ in the form $\left(x + \dfrac{b}{2}\right)^2 - \left(\dfrac{b}{2}\right)^2 + c = 0$
$(x + 3)^2 - 9 + 4 = 0$	
$(x + 3)^2 - 5 = 0$	2 Simplify.
$(x + 3)^2 = 5$	3 Rearrange the equation to work out x. First, add 5 to both sides.
$x + 3 = \pm\sqrt{5}$	4 Square root both sides. Remember that the square root of a value gives two answers.
$x = \pm\sqrt{5} - 3$	5 Subtract 3 from both sides to solve the equation.
So $x = -\sqrt{5} - 3$ or $x = \sqrt{5} - 3$	6 Write down both solutions.

Example 6   Solve $2x^2 - 7x + 4 = 0$. Give your solutions in surd form.

$2x^2 - 7x + 4 = 0$	1 Before completing the square write $ax^2 + bx + c$ in the form $a\left(x^2 + \dfrac{b}{a}x\right) + c$
$2\left(x^2 - \dfrac{7}{2}x\right) + 4 = 0$	
$2\left[\left(x - \dfrac{7}{4}\right)^2 - \left(\dfrac{7}{4}\right)^2\right] + 4 = 0$	2 Now complete the square by writing $x^2 - \dfrac{7}{2}x$ in the form $\left(x + \dfrac{b}{2a}\right)^2 - \left(\dfrac{b}{2a}\right)^2$
$2\left(x - \dfrac{7}{4}\right)^2 - \dfrac{49}{8} + 4 = 0$	3 Expand the square brackets.
$2\left(x - \dfrac{7}{4}\right)^2 - \dfrac{17}{8} = 0$	4 Simplify.

$$2\left(x - \frac{7}{4}\right)^2 = \frac{17}{8}$$	**5** Rearrange the equation to work out x. First, add $\frac{17}{8}$ to both sides.
$$\left(x - \frac{7}{4}\right)^2 = \frac{17}{16}$$	**6** Divide both sides by 2.
$$x - \frac{7}{4} = \pm\frac{\sqrt{17}}{4}$$	**7** Square root both sides. Remember that the square root of a value gives two answers.
$$x = \pm\frac{\sqrt{17}}{4} + \frac{7}{4}$$	**8** Add $\frac{7}{4}$ to both sides.
So $x = \frac{7}{4} - \frac{\sqrt{17}}{4}$ or $x = \frac{7}{4} + \frac{\sqrt{17}}{4}$	**9** Write down both the solutions.

Practice

3 Solve by completing the square.

    a $x^2 - 4x - 3 = 0$ **b** $x^2 - 10x + 4 = 0$

    c $x^2 + 8x - 5 = 0$ **d** $x^2 - 2x - 6 = 0$

    e $2x^2 + 8x - 5 = 0$ **f** $5x^2 + 3x - 4 = 0$

4 Solve by completing the square.

    a $(x - 4)(x + 2) = 5$

    b $2x^2 + 6x - 7 = 0$

    c $x^2 - 5x + 3 = 0$

Hint

Get all terms onto one side of the

Solving quadratic equations by using the formula*

Key points

- Any quadratic equation of the form $ax^2 + bx + c = 0$ can be solved using the formula
  $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
- If $b^2 - 4ac$ is negative then the quadratic equation does not have any real solutions.
- It is useful to write down the formula before substituting the values for a, b and c.

Examples

Example 7 Solve $x^2 + 6x + 4 = 0$. Give your solutions in surd form.

$a = 1, b = 6, c = 4$ $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	**1** Identify a, b and c and write down the formula. Remember that $-b \pm \sqrt{b^2 - 4ac}$ is all over $2a$, not just part of it.
$x = \dfrac{-6 \pm \sqrt{6^2 - 4(1)(4)}}{2(1)}$ $x = \dfrac{-6 \pm \sqrt{20}}{2}$	**2** Substitute $a = 1$, $b = 6$, $c = 4$ into the formula. **3** Simplify. The denominator is 2, but this is only because $a = 1$. The denominator will not always be 2.
$x = \dfrac{-6 \pm 2\sqrt{5}}{2}$	**4** Simplify $\sqrt{20}$. $\sqrt{20} = \sqrt{4 \times 5} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$
$x = -3 \pm \sqrt{5}$	**5** Simplify by dividing numerator and denominator by 2.
So $x = -3 - \sqrt{5}$ or $x = \sqrt{5} - 3$	**6** Write down both the solutions.

Example 8   Solve $3x^2 - 7x - 2 = 0$. Give your solutions in surd form.

$a = 3, b = -7, c = -2$ $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$	**1** Identify a, b and c, making sure you get the signs right and write down the formula. Remember that $-b \pm \sqrt{b^2 - 4ac}$ is all over $2a$, not just part of it.
$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(-2)}}{2(3)}$$	**2** Substitute $a = 3$, $b = -7$, $c = -2$ into the formula.
$$x = \frac{7 \pm \sqrt{73}}{6}$$ So $x = \dfrac{7 - \sqrt{73}}{6}$ or $x = \dfrac{7 + \sqrt{73}}{6}$	**3** Simplify. The denominator is 6 when $a = 3$. A common mistake is to always write a denominator of 2. **4** Write down both the solutions.

Practice

5   Solve, giving your solutions in surd form.
   a $3x^2 + 6x + 2 = 0$ **b** $2x^2 - 4x - 7 = 0$

6   Solve the equation $x^2 - 7x + 2 = 0$

   Give your solutions in the form $\dfrac{a \pm \sqrt{b}}{c}$, where a, b and c are integers.

7   Solve $10x^2 + 3x + 3 = 5$
   Give your solution in surd form.

> **Hint**
>
> Get all terms onto one side of the equation.

Extend

8   Choose an appropriate method to solve each quadratic equation, giving your answer in surd form when necessary.
   a $4x(x - 1) = 3x - 2$
   b $10 = (x + 1)^2$
   c $x(3x - 1) = 10$

Sketching quadratic graphs

Key points

- The graph of the quadratic function $y = ax^2 + bx + c$, where $a \neq 0$, is a curve called a parabola.
- Parabolas have a line of symmetry and a shape as shown.

for $a > 0$ for $a < 0$

- To sketch the graph of a function, find the points where the graph intersects the axes.
- To find where the curve intersects the y-axis substitute $x = 0$ into the function.
- To find where the curve intersects the x-axis substitute $y = 0$ into the function.
- At the turning points of a graph the gradient of the curve is 0 and any tangents to the curve at these points are horizontal.
- To find the coordinates of the maximum or minimum point (turning points) of a quadratic curve (parabola) you can use the completed square form of the function.

Examples

Example 1   Sketch the graph of $y = x^2$.

	The graph of $y = x^2$ is a parabola. When $x = 0$, $y = 0$. $a = 1$ which is greater than zero, so the graph has the shape:

Example 2   Sketch the graph of $y = x^2 - x - 6$.

When $x = 0$, $y = 0^2 - 0 - 6 = -6$ So the graph intersects the y-axis at $(0, -6)$ When $y = 0$, $x^2 - x - 6 = 0$ $(x + 2)(x - 3) = 0$ $x = -2$ or $x = 3$ So, the graph intersects the x-axis at $(-2, 0)$ and $(3, 0)$	**1** Find where the graph intersects the y-axis by substituting $x = 0$. **2** Find where the graph intersects the x-axis by substituting $y = 0$. **3** Solve the equation by factorising. **4** Solve $(x + 2) = 0$ and $(x - 3) = 0$. **5** $a = 1$ which is greater than zero, so the graph has the shape:

(continued on next page)

$$x^2 - x - 6 = \left(x - \frac{1}{2}\right)^2 - \frac{1}{4} - 6$$

$$= \left(x - \frac{1}{2}\right)^2 - \frac{25}{4}$$

When $\left(x - \frac{1}{2}\right)^2 = 0$, $x = \frac{1}{2}$ and

$y = -\frac{25}{4}$, so the turning point is at the

point $\left(\frac{1}{2}, -\frac{25}{4}\right)$

$(\frac{1}{2}, -6\frac{1}{4})$

6 To find the turning point, complete the square.

7 The turning point is the minimum value for this expression and occurs when the term in the bracket is equal to zero.

Practice

1 Sketch the graph of $y = -x^2$.

2 Sketch each graph, labelling where the curve crosses the axes.

    a $y = (x + 2)(x - 1)$ **b** $y = x(x - 3)$ **c** $y = (x + 1)(x + 5)$

3 Sketch each graph, labelling where the curve crosses the axes.

    a $y = x^2 - x - 6$ **b** $y = x^2 - 5x + 4$ **c** $y = x^2 - 4$

    d $y = x^2 + 4x$ **e** $y = 9 - x^2$ **f** $y = x^2 + 2x - 3$

4 Sketch the graph of $y = 2x^2 + 5x - 3$, labelling where the curve crosses the axes.

Extend

5 Sketch each graph. Label where the curve crosses the axes and write down the coordinates of the turning point.

    a $y = x^2 - 5x + 6$ **b** $y = -x^2 + 7x - 12$ **c** $y = -x^2 + 4x$

6 Sketch the graph of $y = x^2 + 2x + 1$. Label where the curve crosses the axes and write down the equation of the line of symmetry.

Quadratic inequalities

Key points

- First replace the inequality sign by = and solve the quadratic equation.
- Sketch the graph of the quadratic function.
- Use the graph to find the values which satisfy the quadratic inequality.

Examples

Example 1   Find the set of values of x which satisfy $x^2 + 5x + 6 > 0$

$x^2 + 5x + 6 = 0$ $(x + 3)(x + 2) = 0$ $x = -3$ or $x = -2$ It is above the x-axis where $x^2 + 5x + 6 > 0$ This part of the graph is not needed as this is where $x^2 + 5x + 6 < 0$ $x < -3$ or $x > -2$	**1** Solve the quadratic equation by factorising. **2** Sketch the graph of $y = (x + 3)(x + 2)$ **3** Identify on the graph where $x^2 + 5x + 6 > 0$, i.e. where $y > 0$ **4** Write down the values which satisfy the inequality $x^2 + 5x + 6 > 0$

Example 2   Find the set of values of x which satisfy $x^2 - 5x \leq 0$

$x^2 - 5x = 0$ $x(x - 5) = 0$ $x = 0$ or $x = 5$ $0 \leq x \leq 5$	**1** Solve the quadratic equation by factorising. **2** Sketch the graph of $y = x(x - 5)$ **3** Identify on the graph where $x^2 - 5x \leq 0$, i.e. where $y \leq 0$ **4** Write down the values which satisfy the inequality $x^2 - 5x \leq 0$

Example 3   Find the set of values of x which satisfy $-x^2 - 3x + 10 \geq 0$

$-x^2 - 3x + 10 = 0$ $(-x + 2)(x + 5) = 0$ $x = 2$ or $x = -5$	**1** Solve the quadratic equation by factorising.
	2 Sketch the graph of $y = (-x + 2)(x + 5) = 0$
	3 Identify on the graph where $-x^2 - 3x + 10 \geq 0$, i.e. where $y \geq 0$
$-5 \leq x \leq 2$	**3** Write down the values which satisfy the inequality $-x^2 - 3x + 10 \geq 0$

Practice

1   Find the set of values of x for which $(x + 7)(x - 4) \leq 0$

2   Find the set of values of x for which $x^2 - 4x - 12 \geq 0$

3   Find the set of values of x for which $2x^2 - 7x + 3 < 0$

4   Find the set of values of x for which $4x^2 + 4x - 3 > 0$

5   Find the set of values of x for which $12 + x - x^2 \geq 0$

Extend

Find the set of values which satisfy the following inequalities.

6 $x^2 + x \leq 6$

7 $x(2x - 9) < -10$

8 $6x^2 \geq 15 + x$

Trigonometry in right-angled triangles

Key points

- In a right-angled triangle:
    - the side opposite the right angle is called the hypotenuse
    - the side opposite the angle θ is called the opposite
    - the side next to the angle θ is called the adjacent.

- In a right-angled triangle:
    - the ratio of the opposite side to the hypotenuse is the sine of angle θ, $\sin\theta = \dfrac{\text{opp}}{\text{hyp}}$

    - the ratio of the adjacent side to the hypotenuse is the cosine of angle θ, $\cos\theta = \dfrac{\text{adj}}{\text{hyp}}$

    - the ratio of the opposite side to the adjacent side is the tangent of angle θ, $\tan\theta = \dfrac{\text{opp}}{\text{adj}}$

- If the lengths of two sides of a right-angled triangle are given, you can find a missing angle using the inverse trigonometric functions: $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$.

- The sine, cosine and tangent of some angles may be written exactly.

	0	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	

Examples

Example 1 Calculate the length of side x.
Give your answer correct to 3 significant figures.

$\cos\theta = \dfrac{\text{adj}}{\text{hyp}}$ $\cos 25° = \dfrac{6}{x}$ $x = \dfrac{6}{\cos 25°}$ $x = 6.620\ 267\ 5...$ $x = 6.62$ cm	**1** Always start by labelling the sides. **2** You are given the adjacent and the hypotenuse so use the cosine ratio. **3** Substitute the sides and angle into the cosine ratio. **4** Rearrange to make x the subject. **5** Use your calculator to work out $6 \div \cos 25°$. **6** Round your answer to 3 significant figures and write the units in your answer.

Example 2 Calculate the size of angle x.
Give your answer correct to 3 significant figures.

$\tan\theta = \dfrac{\text{opp}}{\text{adj}}$ $\tan x = \dfrac{3}{4.5}$ $x = \tan^{-1}\left(\dfrac{3}{4.5}\right)$ $x = 33.690\ 067\ 5...$ $x = 33.7°$	**1** Always start by labelling the sides. **2** You are given the opposite and the adjacent so use the tangent ratio. **3** Substitute the sides and angle into the tangent ratio. **4** Use $\tan^{-1}$ to find the angle. **5** Use your calculator to work out $\tan^{-1}(3 \div 4.5)$. **6** Round your answer to 3 significant figures and write the units in your answer.

Example 3   Calculate the exact size of angle x.

	1 Always start by labelling the sides.
$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$ $$\tan x = \frac{\sqrt{3}}{3}$$ $$x = 30°$$	2 You are given the opposite and the adjacent so use the tangent ratio. 3 Substitute the sides and angle into the tangent ratio. 4 Use the table from the key points to find the angle.

Practice

1   Calculate the length of the unknown side in each triangle.
     Give your answers correct to 3 significant figures.

a

b

c

d

e

f

2 Calculate the size of angle x in each triangle.
Give your answers correct to 1 decimal place.

a

5 cm
x
4 cm

b

6.3 cm
7.5 cm
x

c
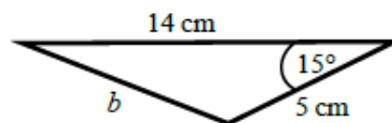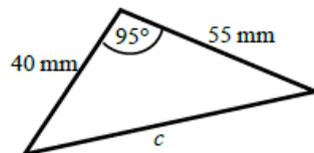

45 mm
42 mm
x

d

4 cm
2.5 cm
x

3 Work out the height of the isosceles triangle.
Give your answer correct to 3 significant figures.

Hint:

Split the triangle into two
right-angled triangles.

55°
55°
8 cm

4 Calculate the size of angle θ.
Give your answer correct to 1 decimal place.

Hint:

First work out the length of the
common side to both triangles,
leaving your answer in surd form.

3 cm
5 cm
θ
7 cm

5 Find the exact value of x in each triangle.

a

2 cm
x
2 cm

b

x
$\sqrt{3}$ cm
60°

c

x
3 cm
6 cm

d

60°
2 cm
x

The cosine rule

Key points

- *a* is the side opposite angle A.
  b is the side opposite angle B.
  c is the side opposite angle C.

- You can use the cosine rule to find the length of a side when two sides and the included angle are given.
- To calculate an unknown side use the formula $a^2 = b^2 + c^2 - 2bc \cos A$.

- Alternatively, you can use the cosine rule to find an unknown angle if the lengths of all three sides are given.
- To calculate an unknown angle use the formula $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$.

Examples

Example 4   Work out the length of side *w*.
Give your answer correct to 3 significant figures.

$a^2 = b^2 + c^2 - 2bc \cos A$	**2** Write the cosine rule to find the side.
$w^2 = 8^2 + 7^2 - 2 \times 8 \times 7 \times \cos 45°$	**3** Substitute the values *a*, *b* and *A* into the formula.
$w^2 = 33.804\,040\,51...$ $w = \sqrt{33.80404051}$ $w = 5.81$ cm	**4** Use a calculator to find w^2 and then *w*. **5** Round your final answer to 3 significant figures and write the units in your answer.

1 Always start by labelling the angles and sides.

Example 5 Work out the size of angle θ.
Give your answer correct to 1 decimal place.

	1 Always start by labelling the angles and sides.
$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$	**2** Write the cosine rule to find the angle.
$\cos\theta = \dfrac{10^2 + 7^2 - 15^2}{2 \times 10 \times 7}$	**3** Substitute the values a, b and c into the formula.
$\cos\theta = \dfrac{-76}{140}$	**4** Use $\cos^{-1}$ to find the angle.
$\theta = 122.878\,349...$	**5** Use your calculator to work out $\cos^{-1}(-76 \div 140)$.
$\theta = 122.9°$	**6** Round your answer to 1 decimal place and write the units in your answer.

Practice

6 Work out the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

a

b

c

d

7 Calculate the angles labelled θ in each triangle.
Give your answer correct to 1 decimal place.

a

9 cm
a
5 cm
12 cm

b

33 mm
38 mm
b
36 mm

c

7.2 cm
c
7.6 cm
13 cm

d

d
8 cm
7.5 cm
11.3 cm

8 **a** Work out the length of WY.
Give your answer correct to
3 significant figures.

b Work out the size of angle WXY.
Give your answer correct to
1 decimal place.

14 cm
X
W
5 cm
8.5 cm
Y
95°
10 cm
Z

The sine rule

Key points

- a is the side opposite angle A.
  b is the side opposite angle B.
  c is the side opposite angle C.

- You can use the sine rule to find the length of a side when its opposite angle and another opposite side and angle are given.

- To calculate an unknown side use the formula $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$.

- Alternatively, you can use the sine rule to find an unknown angle if the opposite side and another opposite side and angle are given.

- To calculate an unknown angle use the formula $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$.

Examples

Example 6   Work out the length of side x.
Give your answer correct to 3 significant figures.

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$	**1** Always start by labelling the angles and sides.
$\dfrac{x}{\sin 36°} = \dfrac{10}{\sin 75°}$	**2** Write the sine rule to find the side.
$x = \dfrac{10 \times \sin 36°}{\sin 75°}$	**3** Substitute the values a, b, A and B into the formula.
$x = 6.09$ cm	**4** Rearrange to make x the subject.
	5 Round your answer to 3 significant figures and write the units in your answer.

Example 7 Work out the size of angle θ.
Give your answer correct to 1 decimal place.

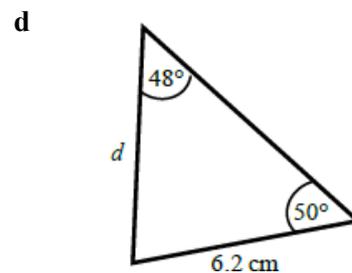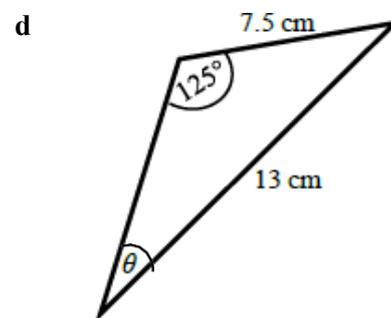|	
$\dfrac{\sin A}{a} = \dfrac{\sin B}{b}$	**2** Write the sine rule to find the angle.
$\dfrac{\sin\theta}{8} = \dfrac{\sin 127°}{14}$	**3** Substitute the values a, b, A and B into the formula.
$\sin\theta = \dfrac{8\times\sin 127°}{14}$	**4** Rearrange to make $\sin\theta$ the subject.
$\theta = 27.2°$	**5** Use $\sin^{-1}$ to find the angle. Round your answer to 1 decimal place and write the units in your answer.

1 Always start by labelling the angles and sides.

Practice

9 Find the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

a

b

c

d

10 Calculate the angles labelled θ in each triangle.
Give your answer correct to 1 decimal place.

a

b

c

d

11 a Work out the length of QS.
Give your answer correct to 3 significant figures.

b Work out the size of angle RQS.
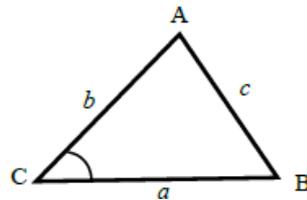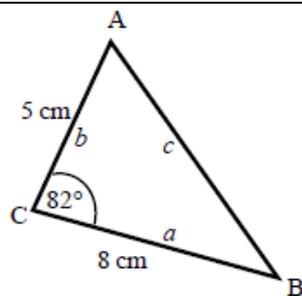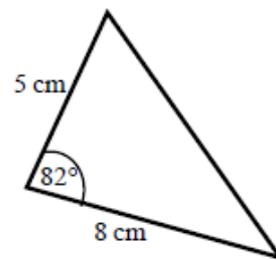Give your answer correct to 1 decimal place.

Areas of triangles

Key points

- *a* is the side opposite angle A.
  b is the side opposite angle B.
  c is the side opposite angle C.
- The area of the triangle is $\frac{1}{2}ab\sin C$.

Examples

Example 8 Find the area of the triangle.

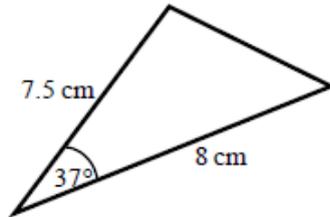|	
Area = $\frac{1}{2}ab\sin C$	**1** Always start by labelling the sides and angles of the triangle.
Area = $\frac{1}{2}\times8\times5\times\sin 82°$	**2** State the formula for the area of a triangle.
	3 Substitute the values of *a*, *b* and *C* into the formula for the area of a triangle.
Area = 19.805 361...	**4** Use a calculator to find the area.
Area = 19.8 cm²	**5** Round your answer to 3 significant figures and write the units in your answer.

Practice

12 Work out the area of each triangle.
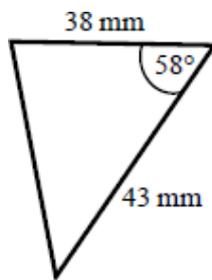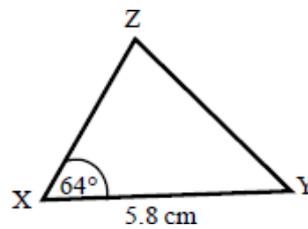Give your answers correct to 3 significant figures.

a

7.5 cm

8 cm

37°

b

5.5 cm

115°

7.5 cm

c

38 mm

58°

43 mm

13 The area of triangle XYZ is 13.3 cm².
Work out the length of XZ.

Hint:

Rearrange the formula to make a side the subject.

Z

X 64°

5.8 cm

Y

Extend

Hint:

For each one, decide whether to use the cosine or sine rule.
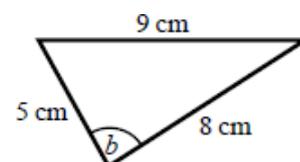
14 Find the size of each lettered angle or side.
Give your answers correct to 3 significant figures.

a

7.5 cm

a

52°

70°

b

9 cm

5 cm

8 cm

b

c

2.5 cm   122°   4 cm

c

d

38 mm   d   20°

95 mm

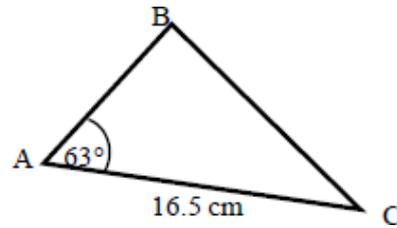**15** The area of triangle ABC is 86.7 cm².
Work out the length of BC.
Give your answer correct to 3 significant figures.

B

A   63°

16.5 cm   C

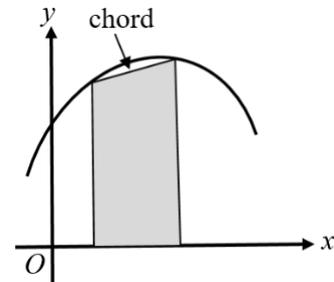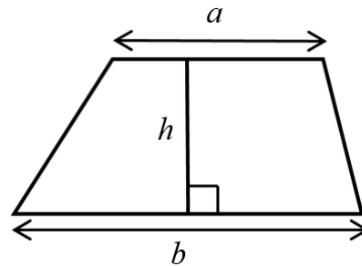# Area under a graph

Key points

- To estimate the area under a curve, draw a chord between the two points you are finding the area between and straight lines down to the horizontal axis to create a trapezium. The area of the trapezium is an approximation for the area under a curve.

- The area of a trapezium = $\frac{1}{2}h(a+b)$

Examples

Example 1   Estimate the area of the region between the curve $y = (3 - x)(2 + x)$ and the *x*-axis from $x = 0$ to $x = 3$. Use three strips of width 1 unit.

x	0	1	2	3
$y = (3 - x)(2 + x)$	6	6	4	0

Trapezium 1:
$a_1 = 6 - 0 = 6$, $b_1 = 6 - 0 = 6$
Trapezium 2:
$a_2 = 6 - 0 = 6$, $b_2 = 4 - 0 = 4$
Trapezium 3:
$a_3 = 4 - 0 = 4$, $a_3 = 0 - 0 = 0$

1 Use a table to record the value of y on the curve for each value of x.

2 Work out the dimensions of each trapezium. The distances between the y-values on the curve and the x-axis give the values for a.

$\frac{1}{2}h(a_1 + b_1) = \frac{1}{2} \times 1(6+6) = 6$ $\frac{1}{2}h(a_2 + b_2) = \frac{1}{2} \times 1(6+4) = 5$ $\frac{1}{2}h(a_3 + b_3) = \frac{1}{2} \times 1(4+0) = 2$ Area = 6 + 5 + 2 = 13 units^2	**3** Work out the area of each trapezium. $h = 1$ since the width of each trapezium is 1 unit. **4** Work out the total area. Remember to give units with your answer.

Example 2   Estimate the shaded area.
Use three strips of width 2 units.

	1 Use a table to record y on the curve for each value of x.

x	4	6	8	10
y	7	12	13	4

2 Use a table to record y on the straight line for each value of x.

x	4	6	8	10
y	7	6	5	4

3 Work out the dimensions of each trapezium. The distances between the y-values on the curve and the y-values on the straight line give the values for a.

Trapezium 1:
$a_1 = 7 - 7 = 0$, $b_1 = 12 - 6 = 6$
Trapezium 2:
$a_2 = 12 - 6 = 6$, $b_2 = 13 - 5 = 8$
Trapezium 3:
$a_3 = 13 - 5 = 8$, $a_3 = 4 - 4 = 0$

$\frac{1}{2}h(a_1 + b_1) = \frac{1}{2} \times 2(0+6) = 6$

$\frac{1}{2}h(a_2 + b_2) = \frac{1}{2} \times 2(6+8) = 14$

$\frac{1}{2}h(a_3 + b_3) = \frac{1}{2} \times 2(8+0) = 8$

4 Work out the area of each trapezium. $h = 2$ since the width of each trapezium is 2 units.

Area = 6 + 14 + 8 = 28 units^2

5 Work out the total area. Remember to give units with your answer.
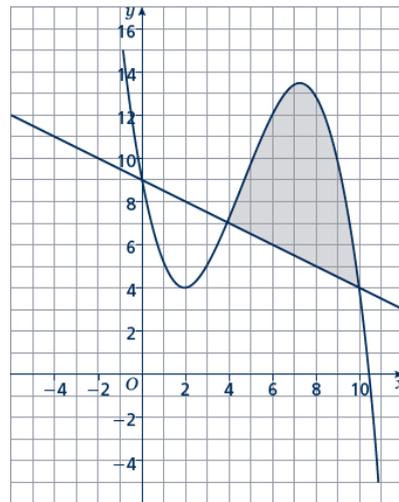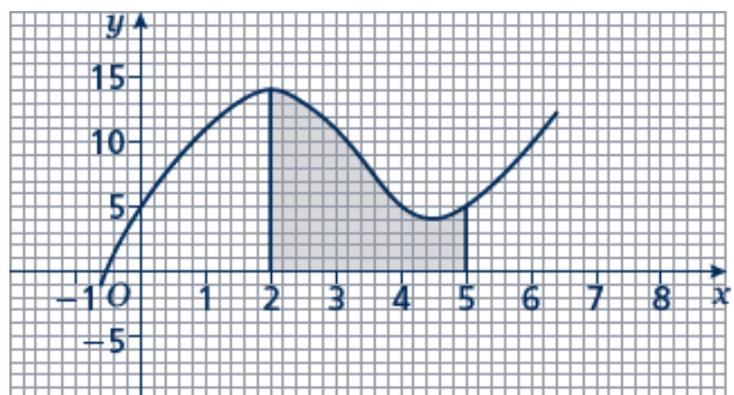
Practice

1 Estimate the area of the region between the curve $y = (5 - x)(x + 2)$ and the x-axis from $x = 1$ to $x = 5$.
Use four strips of width 1 unit.

2 Estimate the shaded area shown on the axes.
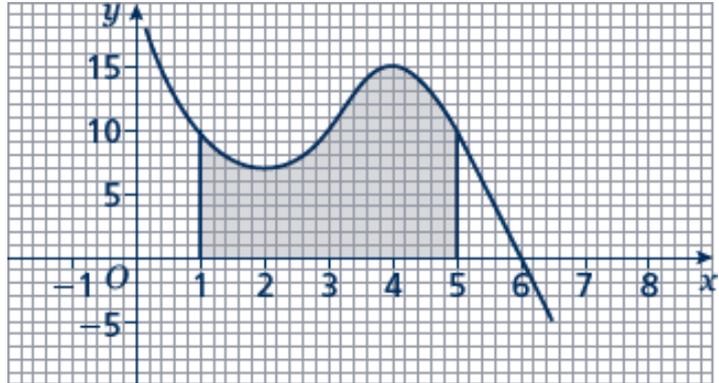Use six strips of width 1 unit.

3 Estimate the area of the region between the curve $y = x^2 - 8x + 18$ and the x-axis from $x = 2$ to $x = 6$.
Use four strips of width 1 unit.

4 Estimate the shaded area.
Use six strips of width $\frac{1}{2}$ unit.

5   Estimate the area of the region between the curve $y = -x^2 - 4x + 5$ and the
x-axis from $x = -5$ to $x = 1$.
Use six strips of width 1 unit.

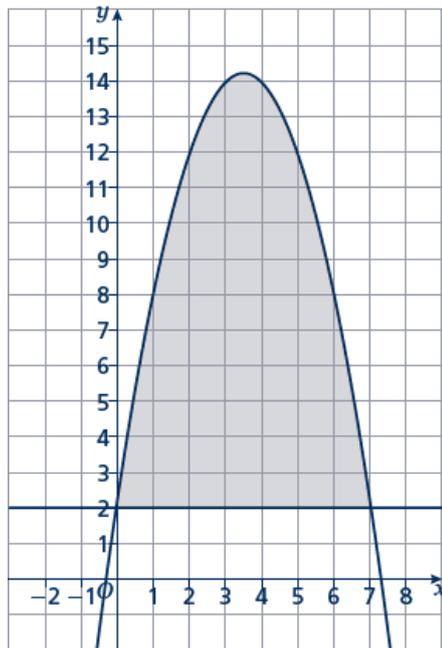**6**   Estimate the shaded area.
Use four strips of equal width.

7   Estimate the area of the region between the curve $y = -x^2 + 2x + 15$ and the
x-axis from $x = 2$ to $x = 5$.
Use six strips of equal width.

8   Estimate the shaded area.
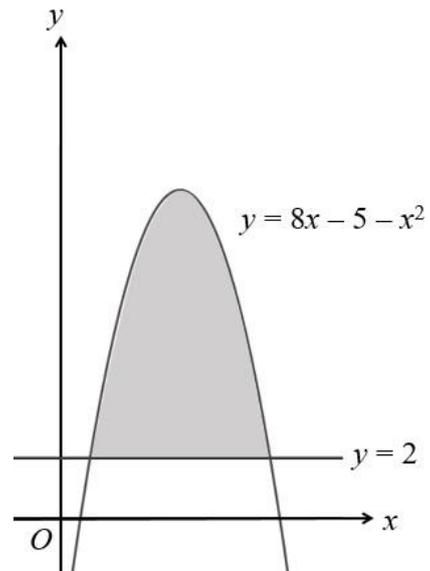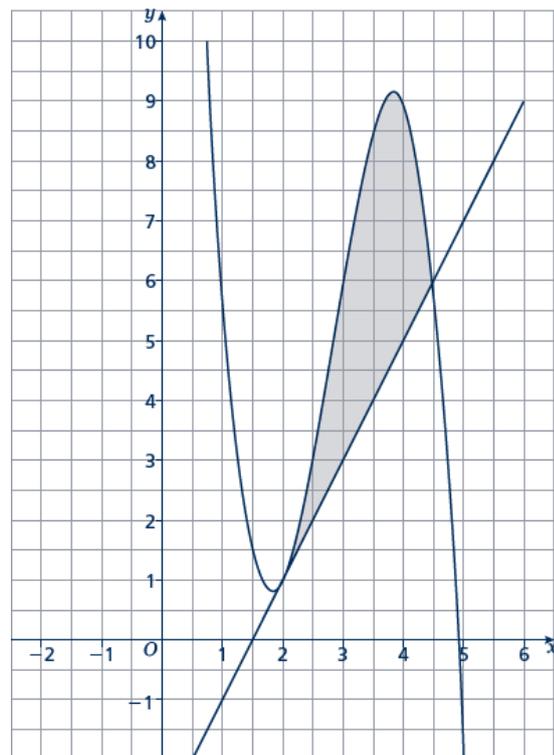Use seven strips of equal width.

Extend

9 The curve $y = 8x - 5 - x^2$ and the line $y = 2$ are shown in the sketch.
Estimate the shaded area using six strips of equal width.

10 Estimate the shaded area using five strips of equal width

? **Example 1** – Simplifying expressions

Simplify these expressions:

a. $x^2 \times x^5$ **b.** $2r^2 \times 3r^3$ **c.** $\dfrac{b^7}{b^4}$ **d.** $6x^5 \div 3x^3$ **e.** $(a^3)^2 \times 2a^2$ **f.** $(3x^2)^3 \div x^4$

? **Example 3** – Simplifying fractions

Solve the following equations:

a. $\dfrac{x^7 + x^4}{x^4}$ **b.** $\dfrac{3x^2 - 6x^5}{2x}$ **c.** $\dfrac{20x^7 - 15x^3}{5x^2}$

? **Example 4** – Expanding double brackets

Expand these expressions and simplify if possible:

a. $(x + 5)(x + 2)$ **b.** $(x - 2y)(x^2 + 1)$ **c.** $(x - y)^2$ **d.** $(x + y)(3x - 2y - 4)$

? **Example 5** – Expanding Trinomials

Expand these expressions and simplify if possible:

a. $x(2x + 3)(x - 7)$ **b.** $x(5x - 3y)(2x - y + 4)$ **c.** $(x - 4)(x + 3)(x + 1)$

? **Example 7** – Factorising quadratics

Factorise:

a. $x^2 - 5x - 6$ **b.** $x^2 + 6x + 8$ **c.** $6x^2 - 11x - 10$ **d.** $x^2 - 25$ **e.** $4x^2 - 9y^2$

? **Example 9** – Simplifying indices

Simplify:

a. $\dfrac{x^3}{x^{-3}}$ **b.** $x^{\frac{1}{2}} \times x^{\frac{3}{2}}$ **c.** $(x^3)^{\frac{2}{3}}$ **d.** $2x^{1.5} \div 4x^{-0.25}$ **e.** $\sqrt[3]{125x^6}$ **f.** $\dfrac{2x^2 - x}{x^5}$

? **Example 10** – Fractional indices

Evaluate:

a. $9^{\frac{1}{2}}$ **b.** $64^{\frac{1}{3}}$ **c.** $49^{\frac{3}{2}}$ **d.** $25^{-\frac{3}{2}}$

? **Example 11** – Indices – problem solving

Given that $y = \dfrac{1}{16}x^2$ express each of the following in the form kx^n, where k and n are constants.

a. $y^{\frac{1}{2}}$ **b.** $4y^{-1}$

? **Example 12** – Simplifying surds

Simplify:

a. $\sqrt{12}$ **b.** $\dfrac{\sqrt{20}}{2}$ **c.** $5\sqrt{6} - 2\sqrt{24} + \sqrt{294}$

? **Example 13** – Expanding brackets and surds

Expand and simplify if possible:

a. $\sqrt{2}\,(5 - \sqrt{3})$ **b.** $(2 - \sqrt{3})\,(5 + \sqrt{3})$